SHOCK STABILITY IN MAGNETOGASDYNAMIC CHANNEL FLOWS

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The question of shock stability in a perfect-gas channel flow was examined in [1] in the one-dimensional approximation under various assumptions: the disturbances are not reflected from the channel exit section, weak shock, etc. The results were found to coincide for two specific forms of the boundary conditions at the channel exit, from which it was concluded that the shock was not sensitive to the exit boundary condition. In [2] the question of shock stability was studied numerically in relation to a conducting-gas flow in a flat channel of constant cross section in the presence of a magnetic field (zero electric field intensity). It was established that the shock stability is significantly affected by the form of the conductivity law. A condition for the limiting regime between the stable and unstable regions was also given for flow with a shock wave. It was assumed that the pressure in the channel exit section is given. In this paper the effect of the exit boundary condition on shock stability in gasdynamic and magnetogasdynamic flows is demonstrated for small magnetic Reynolds numbers. Stability criteria are obtained for shocks near the channel exit for a specific exit condition. The influence of electromagnetic effects (conductivity law, electric load factor) on shock stability is investigated.

1. We will consider in the quasi one-dimensional approximation the nonstationary flow of a perfect inviscid nonheat-conducting gas with electrical conductivity $\sigma = \sigma(p, \rho)$ in a flat channel of arbitrary cross section y(x) in the presence of an electromagnetic field. The flow velocity direction coincides with the direction of the x axis. The upper and lower walls of the channel are conductors at a potential difference 2φ; the external magnetic field of intensity B(x) is directed perpendicular to the plane containing the x axis and the generators of the channel walls.

Assuming that the magnetic Reynolds numbers are small, we write the equations of continuity, motion, and energy

$$
\begin{gathered}
y\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u y) = 0 \\
\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} - \sigma B\left(uB - \frac{\varphi}{y}\right) \\
y\frac{\partial p}{\partial t} + uy\frac{\partial p}{\partial x} + \varkappa p\frac{\partial}{\partial x}(uy) = (\varkappa - 1)\sigma y\left(uB - \frac{\varphi}{y}\right)^2
\end{gathered}
\tag{1.1}
$$

Here ρ is density, u velocity, p gas pressure, and $\varkappa$ the ratio of specific heats. In (1.1) we have used the equation of state for a perfect gas.

Let system (1.1) have a certain stationary $\rho = R(x)$, $u = U(x)$, and $p = P(x)$, containing a shock wave (in the given case a normal shock). The variables on either side of the shock are related by the expressions

$$
\begin{gathered}
\rho^-(\delta - u^-) = \rho^+(\delta - u^+) \\
\rho^-u^-(\delta - u^-) - p^- = \rho^+u^+(\delta - u^+) - p^+ \\
\frac{\varkappa}{\varkappa - 1}\frac{p^-}{\rho^-} + \frac{(\delta - u^-)^2}{2} = \frac{\varkappa}{\varkappa - 1}\frac{p^+}{\rho^+} + \frac{(\delta - u^+)^2}{2}\,.
\end{gathered}
\tag{1.2}
$$

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The superscripts "+" and "−" relate to the parameters behind and in front of the shock, respectively; δ is the shock velocity.

We assume that the supersonic flow preceding the shock wave is undisturbed, with given parameters U_a, R_a, P_a at the channel inlet ($x = x_a$). At the channel outlet at $x = x_b$ the boundary condition must be specified. We write this condition in the form

$$\psi\left(u,\ p,\ \rho,\ \frac{\partial u}{\partial x},\ \ldots,\ \frac{\partial u}{\partial t},\ \ldots\right) = 0 \qquad (x = x_b). \tag{1.3}$$

Here ψ is a known function of its arguments. Condition (1.3) determines the location $x = x_c$ of the shock wave in the channel. In particular, this condition may express the requirement of equal pressures in the channel exit section and in the medium into which the flow expands:

$$p_b = p_\infty .$$

For a sonic exit velocity condition (1.3) has the form

$$\rho_b u_b^2 / \varkappa p_b = 1 .$$

We assume that the subsonic part of the stationary solution between the shock wave and the channel exit section is disturbed. We denote the nonstationary velocity, density, and pressure increments by $u^*(x, t)$, $\rho^*(x, t)$, $p^*(x, t)$, respectively. Assuming that the starred quantities are small and, moreover, that their rate of growth is determined by the factor $\exp \lambda t$ [3], we find the solution of system (1.1) linearized with respect to u^*, ρ^*, p^* in the form

$$\rho^*(x, t) = \rho(x) e^{\lambda t}, \quad u^*(x, t) = u(x) e^{\lambda t} \quad p^*(x, t) = p(x) e^{\lambda t}.$$

Here we have retained the same notation for the velocity, density, and pressure as in (1.1), since system (1.1) will not be employed in what follows.

In order to determine $\rho(x)$, $u(x)$, $p(x)$ we obtain a system of linear ordinary differential equations with coefficients depending on the parameter λ and the coordinate x (through the stationary solution $U(x)$, $R(x)$, $P(x)$)

$$\begin{gathered}
[(Uy)'/y + \lambda]\rho + U\rho' + u(Ry)'/y + Ru' = 0 \\
UU' + B\alpha\sigma_\rho)\rho + (RU' + \sigma B^2 + \lambda)u + RUu' + B\alpha\sigma_p p + p' = 0 \\
-(\varkappa - 1)\alpha^2\sigma_\rho\rho + [P' + \varkappa P y'/y - 2(\varkappa - 1)\sigma B\alpha]u + \varkappa P u' \\
+ [\varkappa(Uy)'/y - (\varkappa - 1)\alpha^2\sigma_p + \lambda]p + Up' = 0.
\end{gathered} \tag{1.4}$$

Here $\alpha = UB - \varphi/y$, σ_p, σ_ρ are the corresponding partial derivatives. A prime denotes the derivative with respect to x.

We obtain the boundary conditions for system (1.4) by linearizing the jump equations (1.2) and condition (1.3)

$$\begin{gathered}
\rho_2 U_2 + R_2 u_2 - D(R_2 - R_1) = 0 \\
\rho_2 U_2^2 + 2R_2 U_2 u_2 + p_2 = \xi R_2 U_2 (U_2 - U_1) y'/y + \xi B(\sigma_2\alpha_2 - \sigma_1\alpha_1) \\
\frac{\varkappa}{\varkappa - 1}\left(\frac{p_2}{R_2} - \frac{P_2}{R_2^2}\rho_2\right) + U_2 u_2 - D(U_2 - U_1) = \xi \frac{B}{R_2}\Phi(\sigma_2\alpha_2 - \sigma_1\alpha_1)
\end{gathered} \tag{1.5}$$

$$a^*(\lambda)\rho_b + b^*(\lambda)u_b + c^*(\lambda)p_b = 0. \tag{1.6}$$

Here $\Phi = \varphi/yBU_2$, ξ is a parameter proportional to the displacement of the shock wave; D is a quantity given by the expression $\delta = De^{\lambda t}$; $a^*(\lambda) = \partial\Psi/\partial\rho$, $b^*(\lambda) = \partial\Psi/\partial u$, $c^*(\lambda) = \partial\Psi/\partial p$ at $x = x_b$; the subscripts 1 and 2 relate to the parameters ahead of and behind the shock. In obtaining relations (1.5) we employed the equations of the stationary solution

$$\frac{U'}{U} = \frac{\varkappa M^2\sigma\alpha\beta}{RU^3(1 - M^2)} - \frac{y'}{y(1 - M^2)}, \qquad \beta = UB - \frac{\varkappa - 1}{\varkappa}\varphi/y$$

$$P' = -RUU' - \sigma B\alpha, \qquad R'/R = -U'/U - y'/y, \qquad M^2 = RU^2/\varkappa P$$

and also took into account the fact that the shock moves.

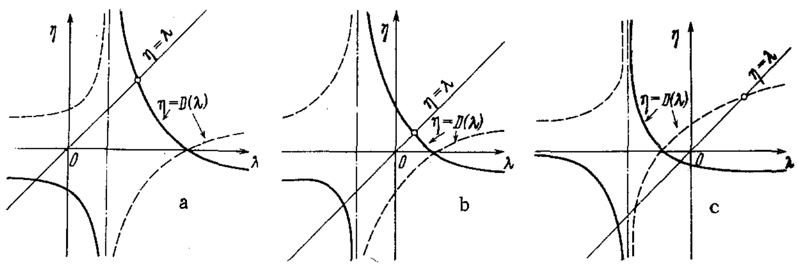

Fig. 1

The solution of system (1.4) depends linearly on the three constants c^1, c^2, c^3, whose equations are obtained by substituting the solution of system (1.4) in boundary conditions (1.5) and (1.6), which must be supplemented by the obvious equation

$$D(\lambda) = \xi\lambda \tag{1.7}$$

The equations for finding ξ, c^1, c^2, c^3 and D constitute a system of five linear homogeneous algebraic equations with a nontrivial solution only at those values of λ that make the determinant of the system vanish. The roots of the determinant having positive real parts give solutions that increase without bound, which indicates the instability of the corresponding stationary solution.

In order to solve the shock-stability problem, it is sufficient to find from Eqs. (1.5) and (1.6) the dependence of the shock velocity on λ and to find the value of λ from Eq. (1.7).

2. We assume that boundary condition (1.6) is satisfied so close to the shock that there is no significant change in the solution on the intervening distance. In this case the effect of different boundary conditions on shock stability can be quite simply demonstrated.

With this assumption Eq. (1.6) has the form

$$a^*(\lambda)\rho_2 + b^*(\lambda) u_2 + c^*(\lambda)\, p_2 = 0 \tag{2.1}$$

We introduce the notation

$$\begin{gathered} z = R_1 / R_2 = U_2 / U_1, \quad \Sigma = \sigma_1 / \sigma_2; \quad K = B(\sigma_2\alpha_2 - \sigma_1\alpha_1) / R_2 \\ = \sigma_2 U_2 B^2 / R_2 \, [\Phi(\Sigma - 1) + 1 - \Sigma/z], \quad a(\lambda) = R_2 a^*(\lambda), \quad b(\lambda) = U_2 b^*(\lambda), \\ c(\lambda) = R_2 U_2^2 c^*(\lambda) \quad f = 1 + (\varkappa - 1)M_2^2 / \varkappa M_2^2, \quad M^2 = RU^2 / \varkappa P \end{gathered}$$

Assuming that the solution between the shock and the channel exit section is equal to $\rho(x) = \rho_2$, $u(x) = u_2$, $p(x) = p_2$, from (1.5) and (2.1) we find

$$\begin{aligned} D(\lambda) = \xi \Big\{ & \frac{\varkappa}{\varkappa - 1} U_2^2 \frac{(z-1)}{z} \frac{y'}{y} [a(\lambda) - b(\lambda) + fc(\lambda)] + \left(\frac{\varkappa}{\varkappa - 1} - \Phi\right) \Big[a(\lambda) \\ & - b(\lambda) + c(\lambda) \left(\frac{\varkappa}{\varkappa - 1} f - \Phi\right) \Big/ \left(\frac{\varkappa}{\varkappa - 1} - \Phi\right)\Big]\Big\} : \left\{\frac{2U_2(1-z)}{(\varkappa - 1)z}\left[\frac{a(\lambda)}{M_1^2} - b(\lambda)\frac{M_1^2 + 1}{2M_1^2} + c(\lambda)\right]\right\} \end{aligned} \tag{2.2}$$

Substituting (2.2) in (1.7), we find the values of λ.

We will examine several different cases.

Case 1. We assume that there is no electromagnetic field, i.e., that the flow is purely gasdynamic. In this case expression (2.2) takes the form

$$D(\lambda) = -\varkappa U_2 M_1^2 \frac{y'}{y} \frac{a(\lambda) - b(\lambda) + fc(\lambda)}{2a(\lambda) - (M_1^2 + 1)\, b(\lambda) + 2M_1^2 c(\lambda)} \xi \tag{2.3}$$

It follows from (2.3) that if $a(\lambda)$ and $c(\lambda)$ are of the same sign, and the sign of $b(\lambda)$ is opposite to that of $c(\lambda)$ at any $\lambda > 0$, then the gas at the channel exit does positive work $(u_2 p_2 > 0)$, since $u_2 = -c/bp_2 - a/b\rho_2$ and the perturbations u_2 and p_2 are of the same sign. In this case the sign of $D(\lambda)$ and hence that of λ (1.7) is determined by the cofactor y'/y. At $y' > 0$, i.e., in the expanding part of the channel, the shock is stable;

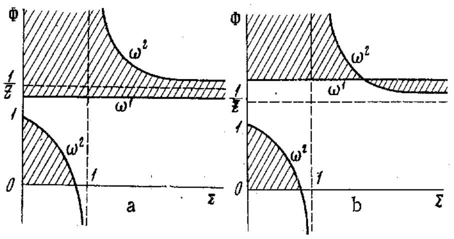

Fig. 2

at $y' < 0$ in the contracting part of the channel the shock is unstable. This result also applies when only the pressure ($c(\lambda) \neq 0$), density ($a(\lambda) \neq 0$) or velocity ($b(\lambda) \neq 0$) is fixed at the exit. On this basis it was concluded in [1] that the shock is indifferent to the boundary condition on the right.

However, if condition (2.1) is such that the coefficients $b(\lambda)$ and $c(\lambda)$ are of the same sign (let $a(\lambda = 0)$, i.e., positive work is done on the gas, then, as is easily seen from (2.3), anything is possible, depending on the behavior of the ratio $b(\lambda)/c(\lambda)$. For example, let the quantity $b(\lambda)/c(\lambda)$ increase monotonically with increase in λ. Then, if at $\lambda = 0$ the numerator and denominator of $D(\lambda)$ are positive (Fig. 1a) or the numerator is positive and the denominator negative (Fig. 1b), then in the expanding channel ($y' > 0$) the shock is unstable, while at $y' < 0$ it is stable. If, however, at $\lambda = 0$ the numerator and denominator are negative, then the shock is stable at $y' > 0$ and unstable at $y' < 0$ (Fig. 1c).

In Fig. 1a, b, c we have graphically represented the result of substituting (2.3) with $a(\lambda) = 0$ and sign $b(\lambda) =$ sign $c(\lambda)$ in Eq. (1.7). The solid lines give $D(\lambda)$ at $y' > 0$, the dashed lines $D(\lambda)$ at $y' < 0$. The intersection of the curve $\eta = D(\lambda)$ and the straight line $\eta = \xi\lambda$ at $\lambda > 0$ (in Fig. 1 a circle has been drawn around these points) gives an idea of the conditions under which the shock is unstable.

Case 2. Let the electromagnetic field be given. In order to demostrate the influence of electromagnetic effects, we assume that the pressure is given at the exit. Then the denominator in (2.2) is positive and equal to $2U_2^3R_2(1-z)/(\varkappa-1)z$. The sign of $D(\lambda)$ and hence λ is determined by the sign of the numerator of $D(\lambda)$. For stability the condition

$$\frac{\varkappa}{\varkappa-1}\frac{(z-1)}{z}U_2^2 f\frac{y'}{y} + K\left(\frac{\varkappa}{\varkappa-1}f - \Phi\right) < 0 \tag{2.4}$$

must be satisfied.

For a flow with constant electrical conductivity ($\sigma =$ const), the quantity $K = \sigma B^2 U_2 R_2^{-1}(1-1/z) < 0$ and the stability condition take the simple form

$$\Phi < [1 + 1/(\varkappa-1)M_2^2]\left(1 + \frac{y' R_2 U_2}{y\sigma B^2}\right). \tag{2.5}$$

It follows, in particular, from (2.5) that at $\sigma =$const in the power-generating regime ($\Phi > 1$) in a straight or expanding channel the shock wave is stable. The term with y'/y on the right side of inequality (2.5) shows that with expansion of the channel ($y' > 0$) the range of Φ on which the shock is stable increases, whereas with contraction of the channel it decreases. As the magnetogasdynamic interaction parameter ($N = \sigma B^2 y/R_2U_2$) increases, the term containing y'/y decreases as compared with unity, and at large values of N this term can be neglected.

There are two possibilities corresponding to the case $\sigma \neq$ const: $\sigma_1/\sigma_2 < 1$, i.e., behind the shock the conductivity increases; $\sigma_1/\sigma_2 > 1$, i.e., behind the shock the conductivity falls.

At $\sigma \neq$ const the shock stability condition takes the form

$$\begin{gathered}\omega^2 < \Phi < \omega^1 \quad \text{at} \quad \Sigma < 1\\ \Phi < \min(\omega^1, \omega^2),\ \Phi > \max(\omega^1, \omega^2) \quad \text{at} \quad \Sigma > 1\\ \omega^{1,2} = \{(1-\Sigma)(1+1/(\varkappa-1)M_2^2) + 1 - \Sigma/z \pm \{[(1-\Sigma)(1+1/(\varkappa-1)\\ \times M_2^2) - (1-\Sigma/z)]^2 + 4(1-\Sigma)(1-z)(1+1/(\varkappa-1)M_2^2)\frac{y' R_2 U_2}{y\sigma_2 B^2 z}\}^{1/2}\} \times [2(1-\Sigma)]^{-1}\end{gathered} \tag{2.6}$$

In a straight channel or at large values of the parameter N, when the term with y' can be neglected, condition (2.6) takes a simpler form. In this case $\omega^1 = 1 + 1/(\varkappa-1)M_2^2$, $\omega^2 = (1-\Sigma/z)(1-\Sigma)^{-1}$. The regions of instability for various values of Σ at $y' = 0$ are represented schematically in Fig. 2a, b. The regions of instability are shaded. The solid lines represent $\Phi = \omega^1$, $\Phi = \omega^2$. The ratio $z = U_2/U_1$ is fixed. It is clear from Fig. 2 that the shocks are stable for any Σ at $1 < \Phi < \omega^1$ (Fig. 2a) or at $1 < \Phi < 1/z$ (Fig. 2b) depending on the relationship between the numbers ω^1 and $1/z$. As $\Sigma \to 1$, as may be seen from Fig. 2, the region of stability with respect to Φ is determined in accordance with (2.5). At $y' > 0$ the region of stability with respect to Φ expands, while at $y' < 0$ the region of stability diminishes.

We present certain numerical examples of ω^1, ω^2 for various laws $\sigma = \sigma(p, \rho)$, $y' = 0$ and various shock intensities.

$\varkappa$	M_1^2	σ_1/σ_2	Stability criterion from (2.6)
		For $\sigma \sim P/R$	
1.2	1.5	0.92	$0 \leqslant \Phi < 8.396$
	10	0.508	$0 \leqslant \Phi < 30.76$
	100	0.092	$0.091 < \Phi < 55.64$
$5/3$	1.5	0.82	$0 \leqslant \Phi < 3.169$
	10	0.25	$0.3 < \Phi < 6.65$
	100	0.031	$0.9 < \Phi < 8.299$
		For $\sigma \sim (P/R)^2$	
1.2	1.5	0.86	$0 \leqslant \Phi < 8.396$
	100	0.0084	$0.92 < \Phi < 55.64$
$5/3$	1.5	0.67	$0.31 < \Phi < 3.169$
	10	0.063	$0.86 < \Phi < 6.65$
	100	0.00096	$0.99 < \Phi < 8.299$
		For $\sigma \sim P^{-1/2} \exp(P/R)$	
1.2	1.5	1.15	$\Phi > 8.396$, $\Phi < 4.31$
	5	1.45	$\Phi > 20.69$, $\Phi < 9.49$
	100	0.0005	$0.99 < \Phi < 55.64$
$5/3$	1.5	1.024	$\Phi > 15.21$, $\Phi < 3.169$
	10	0.17	$0.55 < \Phi < 6.65$
	100	0	$1 < \Phi < 8.299$

It is clear from these data that the dependence of the electrical conductivity on the flow parameters has an important effect on the shock stability. Thus, at $\sigma = \text{const}$ in the power generating regime the shock wave is stable at all Φ ensuring that regime, while at $\sigma \sim T^2$ it is stable only on a small interval of variation of Φ. This was previously observed in [4].

We will now show how the stability of a shock wave in a magnetogasdynamic flow is affected by a change in the boundary conditions at the channel exit. Let not the pressure, as assumed in obtaining criteria (2.5), (2.6), but the velocity or density, be given at the exit. In this case in (2.5), (2.6) the term $(1 + 1/(\varkappa - 1)M_2^2)$ must everywhere be replaced by $\varkappa/(\varkappa - 1)$.

We now assume that boundary condition (2.1) does not contain the density, i.e., $a(\lambda) = 0$. For simplicity, let $\sigma = \text{const}$ and $y' = 0$. Then, if the coefficients $b(\lambda)$ and $c(\lambda)$ are opposite in sign, i.e., the gas at the exit does positive work, then the stability criterion has the form

$$\Phi < \frac{\varkappa}{\varkappa - 1} \frac{b(\lambda) - f\,c(\lambda)}{b(\lambda) - c(\lambda)}\,. \tag{2.7}$$

It is clear from a comparison of (2.7) and (2.5) that here the limit of stability with respect to Φ has changed with the boundary condition, whereas for a gasdynamic flow at $u_2 p_2 \geq 0$ the criterion was preserved.

If, however, the coefficients $b(\lambda)$ and $c(\lambda)$ are of the same sign, then there may be a very important change in the stability criterion depending on the behavior of $b(\lambda)$ and $c(\lambda)$. As with the gasdynamic flow, the investigation may be carried out graphically.

3. The preceding investigation is valid for shocks very close to the exit section. In order to explain the effect of the flow behind the shock on its stability we represent the solution of Eqs. (1.4) behind the shock in the form of a series in Δx (Δx is a coordinate reckoned from the undisturbed position of the shock wave). In the investigation we will confine ourselves to terms of the order of Δx, which give an idea of the nature of this effect.

We write the solution of system of equations (1.4) in the form

$$u = u_2 + u_2'\Delta x, \qquad \rho = \rho_2 + \rho_2'\Delta x, \qquad p = p_2 + p_2'\Delta x \, .$$

For simplicity we will consider only one boundary condition at the exit $p_b = 0$. In the approximation adopted we have

$$p_b = p_2 + p_2'\Delta x = 0 \, . \tag{3.1}$$

In this case the expression for $D(\lambda)$ takes the form

$$D(\lambda) = \left\{ L^1 + \frac{\Delta x}{1 - M_2^2}\left[L^2 \frac{y'}{y} + L^3 \frac{\lambda}{U_2} + L^4 \frac{\sigma_2\alpha_2\beta_2}{R_2U_2^3} - \frac{\alpha_2\gamma_2}{R_2U_2^3}(R_2\sigma_{\rho_2}L^5 + R_2U_2^2\sigma_{p_2}L^1) \right.\right.$$
$$\left.\left. - L^5 \frac{\sigma_2 B\Gamma_2}{R_2U_2^2}\right]\right\} : \frac{2U_2(1-z)}{(\varkappa - 1)z}\left\{1 + \frac{\Delta x}{1 - M_2^2}\left[G^1\frac{y'}{y} + G^2\frac{\lambda}{U_2} + G^3\frac{\sigma_2\alpha_2\beta_2}{R_2U_2^3} - \frac{\alpha_2\gamma_2}{R_2U_2^3}\left(\frac{R_2\sigma_{\rho_2}}{M_1^2} + R_2U_2^2\sigma_{p_2}\right) - G^4\frac{\sigma_2B\Gamma_2}{R_2U_2^2}\right]\right\}$$

$$L^1 = \frac{(z-1)}{z}U_2^2\left[1 + \frac{1}{(\varkappa-1)M_2^2}\right]\frac{y'}{y} + \left[1 + \frac{1}{(\varkappa-1)M_2^2} - \Phi\right]K$$

$$L^2 = -\left\{\frac{\varkappa}{\varkappa-1}\left[\frac{(\varkappa-1)M_2^4 + M_2^2 + 1}{1 - M_2^2}\right]U_2^2\frac{(z-1)}{z}\frac{y'}{y} + \frac{\varkappa M_2^4 + 1}{1 - M_2^2}K\left[\frac{(\varkappa-1)M_2^4 + M_2^2 + 1}{(\varkappa M_2^4 + 1)(\varkappa - 1)/\varkappa} - \Phi\right]\right\} \tag{3.2}$$

$$L^3 = \frac{(\varkappa-1)M_2^2 + \varkappa + 1}{(\varkappa - 1)}U_2^2\frac{(z-1)}{z}\frac{y'}{y} + (1 + M_2^2)K\left[\frac{(\varkappa-1)M_2^2 + \varkappa + 1}{(\varkappa-1)(1 + M_2^2)} - \Phi\right]$$

$$L^4 = \frac{\varkappa^2M_2^2(1 + \varkappa M_2^2)}{(\varkappa-1)(1 - M_2^2)}U_2^2\frac{(z-1)}{z}\frac{y'}{y} + \frac{\varkappa(\varkappa+1)M_2^4}{(1 - M_2^2)}K\left[\frac{1 + \varkappa M_2^2}{(\varkappa^2 - 1)M_2^2} - \Phi\right]$$

$$L^5 = \frac{\varkappa}{\varkappa - 1}U_2^2\frac{(z-1)}{z}\frac{y'}{y} + K\left(\frac{\varkappa}{\varkappa-1} - \Phi\right)$$

$$\gamma_2 = U_2B + (\varkappa - 1)M_2^2\alpha_2, \qquad \Gamma_2 = U_2B + (2\varkappa - 1)M_2^2\alpha_2$$

$$G^1 = -\frac{\varkappa M_2^4 + 1}{1 - M_2^2}, \qquad G^2 = \frac{2M_1^2M_2^2 + M_1^2 + 1}{2M_1^2}$$

$$G^3 = \frac{\varkappa M_2^2}{2M_1^2(1 - M_2^2)}[(2\varkappa + 1)M_1^2M_2^2 + M_2^2 + M_1^2 - 1]$$

$$G^4 = \frac{M_1^2 + 1}{2M_1^2} \, .$$

We will again consider several cases.

Case 1. Let there be no electromagnetic field. The expression for $D(\lambda)$ takes the form

$$D(\lambda) = -U_2\frac{1 + (\varkappa-1)M_2^2}{2M_2^2}\frac{y'}{y}\,\frac{1 - \theta^1\Delta x y'/y + \theta^2\Delta x\lambda/U_2}{1 - \theta^3\Delta x y'/y + \theta^4\Delta x\lambda/U_2} \tag{3.3}$$

$$\theta^1 = \frac{\varkappa M_2^2[1 + M_2^2 + (\varkappa-1)M_2^4]}{[1 + (\varkappa-1)M_2^2](1 - M_2^2)^2}, \qquad \theta^2 = \frac{\varkappa + 1 + (\varkappa - 1)M_2^2}{\varkappa(1 - M_2^2)}$$

$$\theta^3 = \frac{1 + \varkappa M_2^4}{(1 - M_2^2)^2}, \qquad \theta^4 = \frac{(M_1^2 + 1)(2M_1^2)^{-1} + M_2^2}{1 - M_2^2} \, .$$

All the coefficients $\theta^{1,2,3,4}$ in (3.3) are positive. At $\Delta x = 0$ we obtain (2.3) with $a = b = 0$ and $c \neq 0$. When $\Delta x \neq 0$ the denominator of (3.3) contains the difference $1 - \theta^3\Delta x y'/y$, which at Mach numbers close to unity and $y' > 0$ may become negative in view of the fact that $\theta^3 \sim (1 - M_2^2)^2$ increases without bound as $M_2 \to 1$, which involves a change in the sign of $D(\lambda)$, since $\theta^1 < \theta^3$, and hence in the sign of λ. Thus, weak shocks may be unstable in the expanding part of the channel under boundary condition (3.1). It should be noted that in view of the smallness of ΔX the term $1 - \theta^2\Delta x y'/y$ indicates only how $D(\lambda)$ tends to behave when the solution of system (1.4) is taken into account with the above accuracy. To improve the accuracy it is necessary to take into account the terms $\sim (\Delta x)^2$, etc.

At $y' < 0$ the terms containing y'/y in the denominator and numerator of expression (3.3) are positive and therefore cannot affect the sign of λ.

Case 2. We assume that the electromagnetic field is given, $y' = 0$, and σ = const. Then

$$D(\lambda) = -\frac{(\varkappa-1)U_2N}{2}\frac{[\varkappa/(\varkappa-1)f-\Phi]+\Delta xN(1-M_2^2)^{-1}\chi^3(\Phi)}{1+\Delta xN(1-M_2^2)^{-1}\chi^2(\Phi)} \tag{3.4}$$

where $N = \sigma B^2/R_2U_2$, and χ^2 (Φ) and χ^3 (Φ) are polynomials of the second and third degrees, respectively, in Φ, whose coefficients are functions of $\varkappa$, M_2^2, and λ and can be obtained from (3.2). An investigation has shown that at small λ and large values of the parameter N the denominator of expression (3.4) may become negative, which leads to a change in the sign of D (λ) and λ.

Unstable regimes occur soonest of all, i.e., at the least values of Δx, near

$$\Phi = \Phi^* = \frac{(2\varkappa-1)[(\varkappa+1)M_1^2M_2^2+M_2^2-1]}{(\varkappa-1)[(2\varkappa+1)M_1^2M_2^2+M_2^2+M_1^2-1]}$$

when the condition $\varkappa/(\varkappa-1)f - \Phi > 0$ is satisfied.

We present certain values of ΔxN.

$\varkappa$	M_1^2	$fx/(\varkappa-1)$	Φ *	ΔxN
1.2	1.1	6.498	3.35	0.002
	1.5	8.39	2.89	0.046
$^5/_3$	1.1	2.65	1.77	0.002
	1.5	2.95	1.58	0.038
2.0	1.1	2.10	1.45	0.001
	1.5	2.43	1.32	0.026

At values of ΔxN greater than those presented the region of instability in the neighborhood of Φ^* increases.

We note that there, too, the term containing Δx in the denominator of expression (3.4) indicates only how D(λ) tends to behave when the flow behind the shock is taken into account.

In conclusion, we note that it would be interesting to investigate the specific form of the boundary condition at the channel exit in some flow studied in the one-dimensional approximation.

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